# Handling Uncertainty in Dynamical Systems and Posing New Questions

AHPCRC RMB Meeting – March 7, 2018

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# OUR PROJECT

- Conducted at the University of Texas at El Paso
- From April 2013 to December 2017

# GENERAL OBJECTIVE

Being able to make sense of dynamical phenomena

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This is relevant to many areas:

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In other words: wouldn't it be nice to be able to predict what could happen?

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  - In this project, we looked at optimization algorithms: regularization in particular (4 conf. + 2 journal articles)

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  - **Trajectories:** e.g., of missiles. What if we could provide an envelope of a missile's trajectory under uncertainty of outside conditions (e.g., weather)?

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- Recomputing the behavior of an unfolding event after unexpected changes
  - How to best **inflect the unfolding of an event** known to lead to an undesired situation?
  - Can we recompute parameters to ensure or avoid a given situation?

And all of these with guarantees.

# HOW WE MET THESE NEW OBJECTIVES

- Handling uncertainty
- Making predictions on unfolding events
- Inflecting unfolding events
- While guaranteeing results

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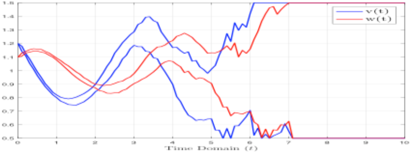
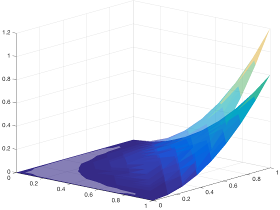
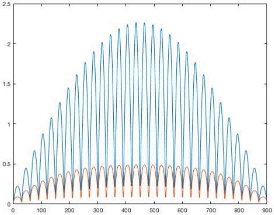
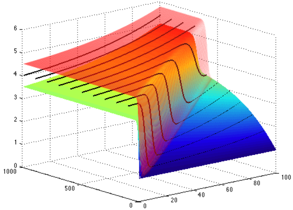
4 conference articles (2 with ARL collaborator), 1 journal article

# UNCERTAINTY: INTERVAL TECHNIQUES







# PREDICTIONS

- **Important note.** Instead of solving: $F_\lambda(x) = 0$, we now solve: $F_{Obs}(\lambda, x \setminus Obs) = 0$, where our observations are uncertain.

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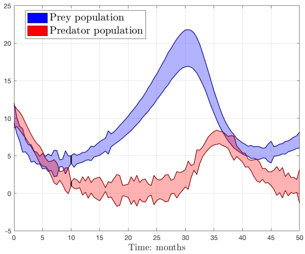


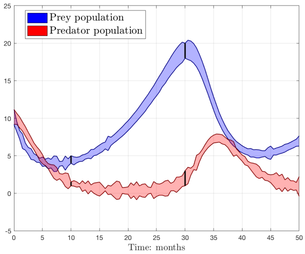


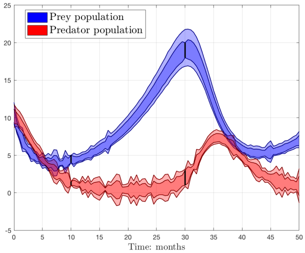

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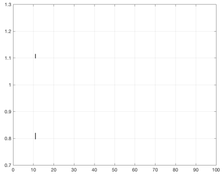

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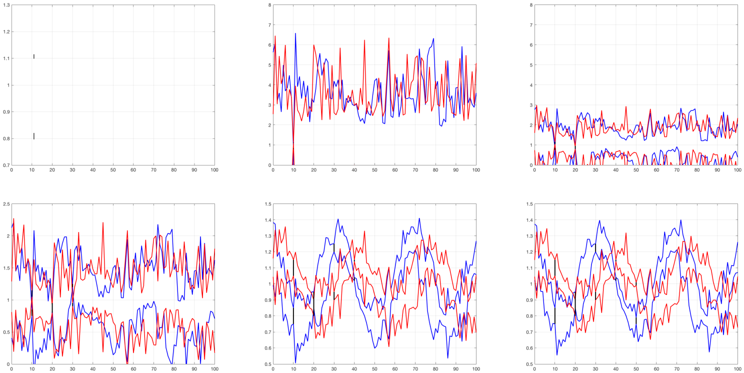

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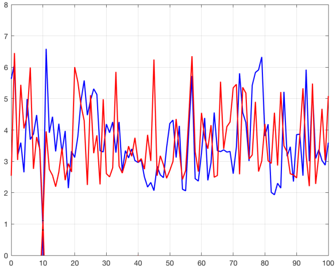


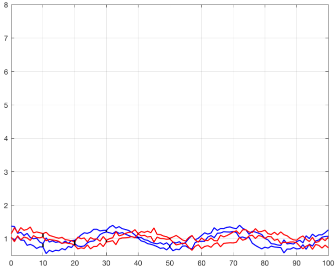

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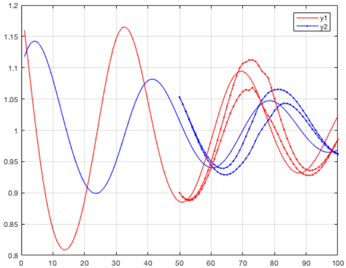


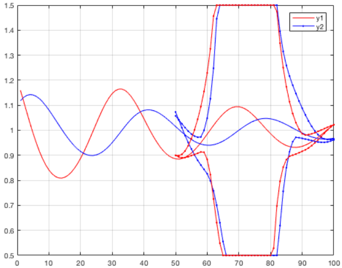

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Some of this work stemmed from collaborations:

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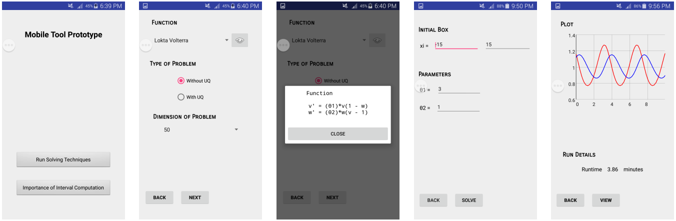
- **Inflections:**
  - From an **APG Open House** visit: how to recompute the load of a helicopter after a hit, to ensure landing in a safe zone.
    $\longrightarrow$ preliminary algorithms to show feasibility of recomputations.

# COLLABORATIONS (CONT'D)

- **Rad Balu – ARL ALC** (2016-2017): explored collaborations about quantum computing; wrote a joint proposal.
- **Simon Su – ARL APG** (2017): our point of contact for our capstone project, a mobile app for handling dynamic systems with uncertainty



# SOFTWARE TRANSFER

- **I-POD package** transferred to Craig Barker's team in 2015.
- **UQ App** available for download.

# VISITS TO ARL

- **Open Campus:** Horacio Florez, post-doc of our team, from 02/2015 to 12/2017, at ARL ALC, Adelphi.
- **Short visits:** to ARL ALC and APG for open houses and other presentations (e.g., poster presentation at June 2017 TAB meeting).

# PUBLICATIONS AND PRESENTATIONS

- **2** edited books
- **5** journal articles
- **15** peer-reviewed conference articles
- **18** conference/workshop presentations
- **5** poster presentations

including: **1** journal article, **2** conference articles, and **1** poster in collaboration with Luis Bravo.

# STUDENTS IMPACTED BY THIS PROJECT

- **2** post-doctoral researchers
- **4** students from my lab (even if not sponsored through this project): 2 Ph.D., 2 UG
- 2016-2017: we identified **19** UTEP students to participate in the AHPCRC Summer Institute: **12** selected

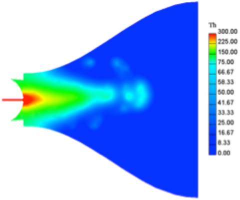
# NEXT STEPS

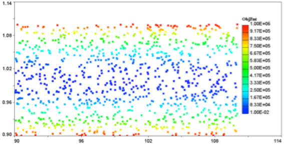
- Model **time uncertainty**
- Beyond uncertainty, handle **erroneous or missing information**

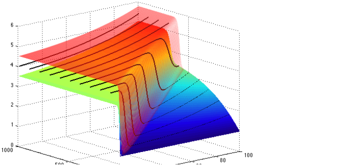
# THANK YOU FOR YOUR ATTENTION

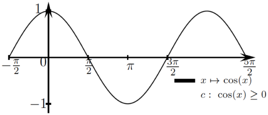
## Any Questions?

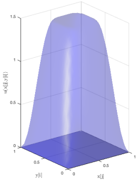
*Below are illustrations of different areas of our work:*

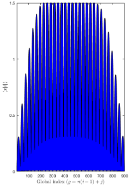


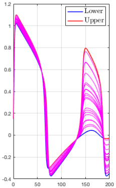



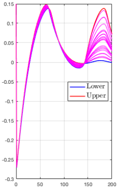








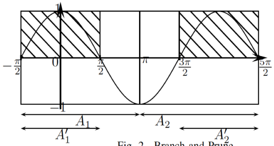




Fig. 2. Branch and Prune