



## Predicting and Controlling the Behavior of Dynamical Systems under Uncertainty

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# RESEARCH INTERESTS

- Optimization and decision under uncertainty

And in particular:

- Optimization: global, nonlinear, under constraints
- Constraint solving: nonlinear, complete and correct, under uncertainty
- Interval computations
- Argumentation networks
- Combinatorial testing

# PROBLEMS OF INTEREST

- Large systems of nonlinear equations – possibly from dynamical systems
- Either full control on these systems for simulations or some uncertain knowledge about it

# RELEVANCE

- Understanding how / under which circumstances-parameters a vehicle can best withstand an underbody blast
- Understanding how a disease spreads depending on, e.g., the number of affected people and the policies put in place
- Understanding how efficient a combustion system is, what performance different mixes of fuel yield
- Identifying parameter values that yield a given outcome
- *etc.*

# TECHNIQUES WE USE

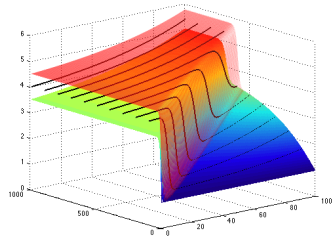
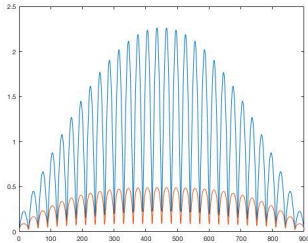
- Interval global optimization under constraints
- Constraint solving techniques
- Reduced-Order Modeling
- Argumentation network

# EXAMPLES

Let's look at a few examples:

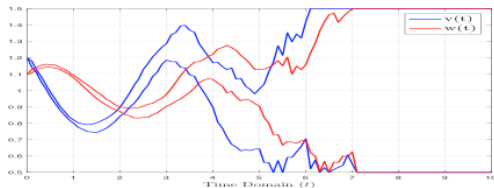
- Simulation of dynamical systems under uncertainty (Interval constraint solving techniques)
- Comparison between Full-Order Model and Reduced-Order Model simulations under uncertainty
- Observation-based prediction of dynamical systems' behavior
- Parameter recomputation under constraints and uncertainty

# INTERVAL CONSTRAINT SOLVING UNDER UNCERTAINTY



# INTERVAL ROM VS. INTERVAL FOM

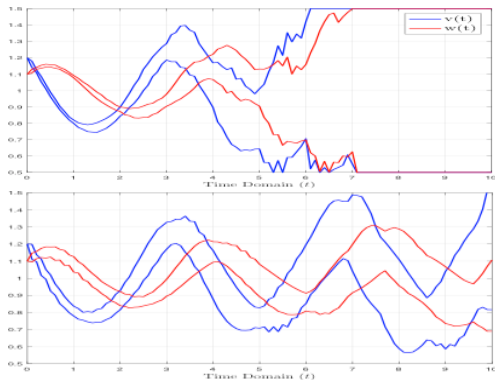
- Lotka-Volterra: FOM (size 200) and with ROM (size 3).





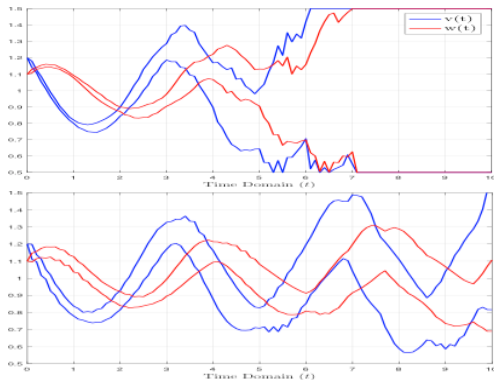
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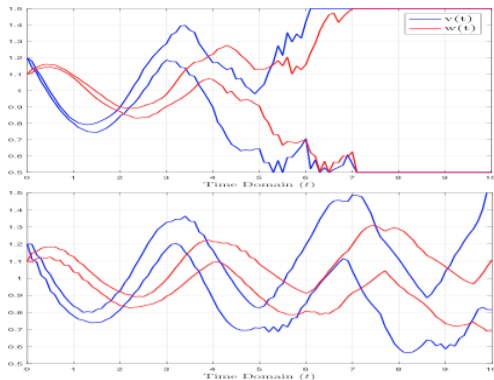
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The runtime is 74,596ms for FOM and 4,616ms for ROM.

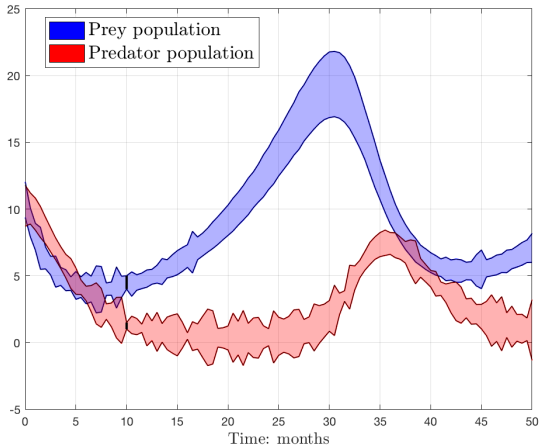
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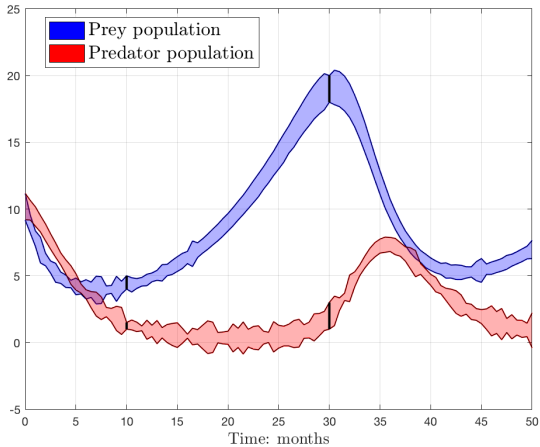


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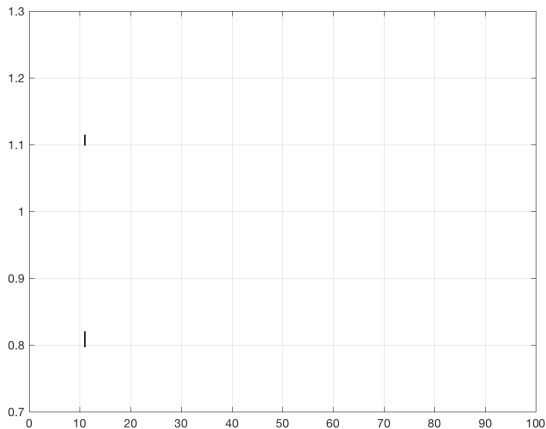
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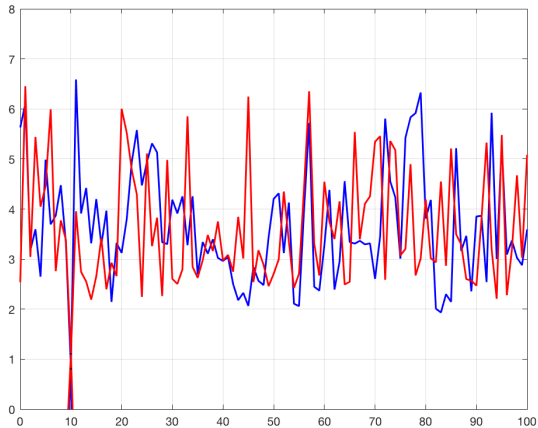


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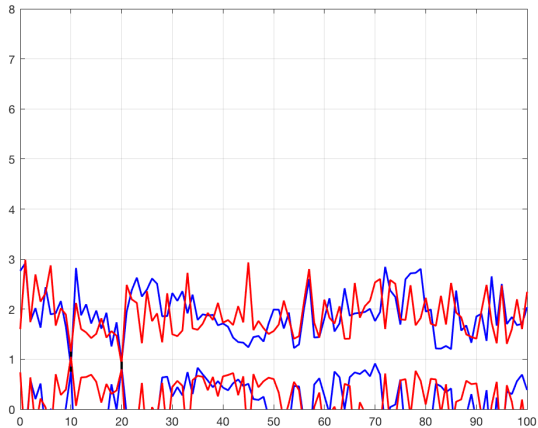


**What we know:** one observation set and  $\theta_1 = \theta_2 = [0, 6]$ .

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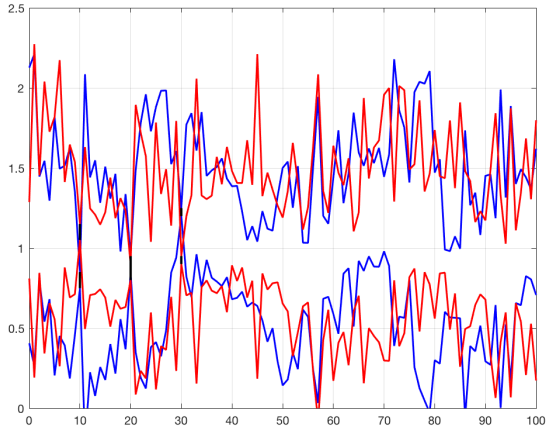


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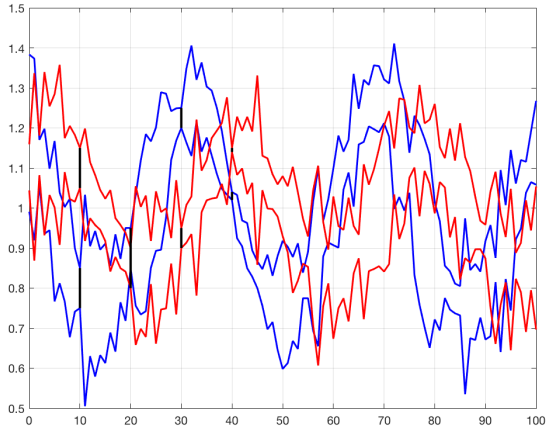




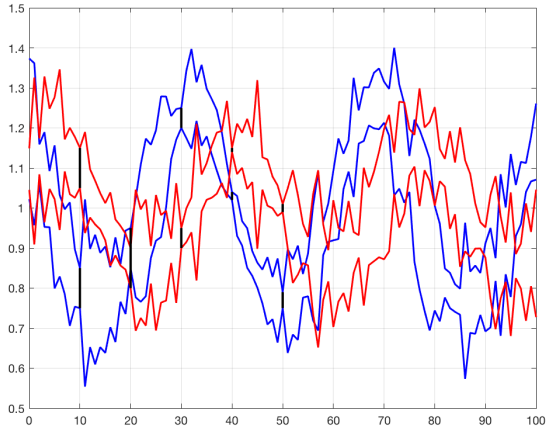
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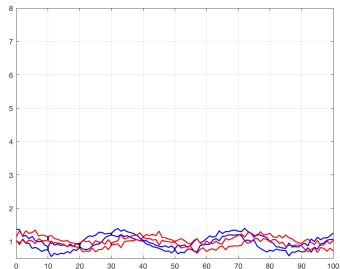
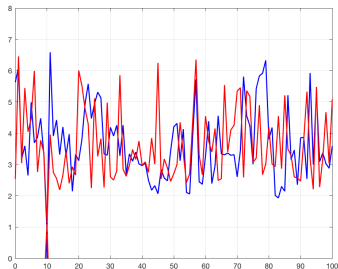


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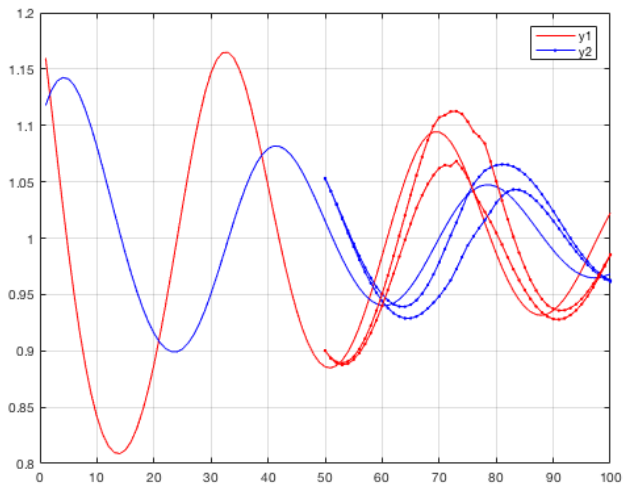


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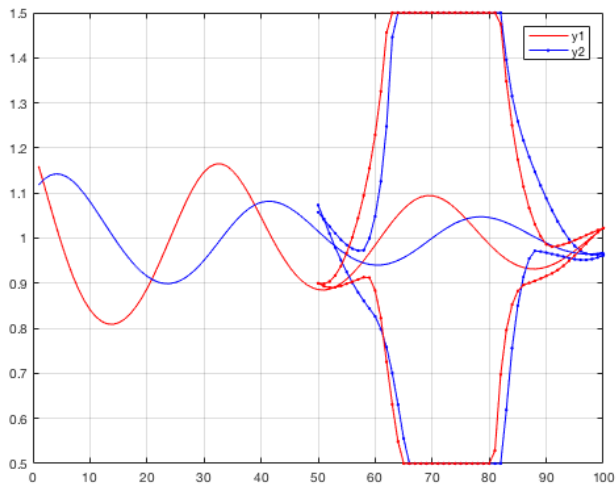
A look at the improvement from start to finish on the same scale:



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  - Recomputation time
  - Control prediction time
- **Applications:**
  - Fuel mix uncertainty
  - Combustion nozzle geometry
  - Problems with discontinuities