Handling Uncertainty in Dynamical Systems and Posing New Questions

AHPCRC RMB Meeting – March 7, 2018

Martine Ceberio, Miguel Argaez, Horacio Florez, Leobardo Valera, Jesus Padilla, Phillip Hassoun

Computer Science Department
The University of Texas at El Paso
mceberio@utep.edu
OUR PROJECT

- Conducted at the University of Texas at El Paso
- From April 2013 to December 2017
GENERAL OBJECTIVE

Being able to make sense of dynamical phenomena
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This is relevant to many areas:

- from understanding how a vehicle can withstand an underbody blast
- to understanding how a disease spreads depending on the number of affected people and the policies put in place for instance,
- to understanding how efficient a combustion system is, what performance different mixes of fuel yield
- etc.
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In other words: wouldn’t it be nice to be able to predict what could happen?
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GENERAL PROBLEM TO BE ADDRESSED

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  - **Complexity:** such problems are likely nonlinear, possibly non-smooth, and yet need to be solved
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  - In this project, we looked at optimization algorithms: regularization in particular (4 conf. + 2 journal articles)
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Fuel combustion: e.g., what decision can be made about the best nozzle geometry if the fuel mix is not known with certainty? Under fuel mix uncertainty, what design could limit pollutant emissions during training but maximize performance on the field?
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- Trajectories: e.g., of missiles. What if we could provide an envelope of a missile’s trajectory under uncertainty of outside conditions (e.g., weather)?
Predicting the future behavior of an unfolding event under observation
ADDITIONAL QUESTIONS / OBJECTIVES

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- **Recomputing** the behavior of an unfolding event after unexpected changes
  - How to best **inflect the unfolding of an event** known to lead to an undesired situation?
  - Can we recompute parameters to ensure or avoid a given situation?

And all of these with **guarantees**.
HOW WE MET THESE NEW OBJECTIVES

- Handling uncertainty
- Making predictions on unfolding events
- Inflecting unfolding events
- While guaranteeing results
Our approach.

Uncertainty

We modeled uncertainty using intervals! interval computations. We had to reconsider optimization algorithms to handle intervals! algorithms based on numerical constraint solving techniques (see poster session this afternoon). We designed a new Finite Element Method technique using intervals for nonlinear functions. Why? Because interval algorithms are reliable: no solution is lost. What this allowed us to do: Simulations with intervals: e.g., uncertainty in initial conditions, in input parameters, etc. Reduced-Order Modeling using interval computations: to handle both the many snapshots and the possible uncertainty in other parameters / constants! new I-POD technique. 4 conference articles (2 with ARL collaborator), 1 journal article.
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UNCERTAINTY: INTERVAL TECHNIQUES
**Important note.** Instead of solving: $F_\lambda(x) = 0$, we now solve: $F_{\text{Obs}}(\lambda, x \setminus \text{Obs}) = 0$, where our observations are uncertain.

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1 conference article, 1 submitted journal article
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Some of this work stemmed from collaborations:

- **Uncertainty:**
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    → integration of interval computations and the design of the I-POD technique.
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**Inflections:**
- From an APG Open House visit: how to recompute the load of a helicopter after a hit, to ensure landing in a safe zone.
  - preliminary algorithms to show feasibility of recomputations.
COLLABORATIONS (CONT'D)


- **Simon Su – ARL APG** (2017): our point of contact for our capstone project, a mobile app for handling dynamic systems with uncertainty

![Mobile Tool Prototype](image1)

![Function](image2)

![Function](image3)

![Initial Box](image4)

![Plot](image5)
SOFTWARE TRANSFER

- **I-POD package** transferred to Craig Barker’s team in 2015.
- **UQ App** available for download.
VISITS TO ARL

- **Open Campus:** Horacio Florez, post-doc of our team, from 02/2015 to 12/2017, at ARL ALC, Adelphi.

- **Short visits:** to ARL ALC and APG for open houses and other presentations (e.g., poster presentation at June 2017 TAB meeting).
2 edited books
5 journal articles
15 peer-reviewed conference articles
18 conference/workshop presentations
5 poster presentations

including: 1 journal article, 2 conference articles, and 1 poster in collaboration with Luis Bravo.
2 post-doctoral researchers

4 students from my lab (even if not sponsored through this project): 2 Ph.D., 2 UG

2016-2017: we identified 19 UTEP students to participate in the AHPCRC Summer Institute: 12 selected
NEXT STEPS

- Model **time uncertainty**
- Beyond uncertainty, handle **erroneous or missing information**
THANK YOU FOR YOUR ATTENTION

Any Questions?

Below are illustrations of different areas of our work: