CAREER: Symbolic-Numeric Constraint-Based Solutions for Real-World Scientific Problems
PI: Martine Ceberio, University of Texas at El Paso (UTEP)

Numerical constraint solving (NCS) techniques have proven to be efficient to solve real problems ranging from electronic circuit to aircraft design. Yet they are under-utilized. The lack of user support of the solvers can partly explain this, but most importantly NCS techniques have isolated themselves into a jargon (constraints) and a schema (propagation/reduction) that disconnect them, respectively, from real problems and real needs (e.g., scalability, flexibility, and distributivity). The **goal of this career plan** is to **make NCS better adapted to real-world needs** while **making it more accessible**. In order to achieve the goal, this project will concentrate on theory and algorithms at the crux of the efficiency, adaptability, and distributivity aspects of problem-solving techniques. In particular, the following research objectives will be pursued:

RO1 To open NCS to novel techniques, improving scalability. The research will be oriented towards designing new algorithms that lie between local consistency techniques and global constraints: symbolic-numeric approaches will be studied from the theoretical point of view, and algorithms will be developed and embedded in a new solving toolbox;

RO2 To assist users facing over-constrained problems. New methods based on the topology of the search space, in terms of satisfaction of the constraints, will be designed to provide users with alternative solutions; and

RO3 To extend the distributed use of NCS to reflect the emerging networked social structure. The structures of social groups and their potential to support distributed systems will be studied, resulting in new and more complete algorithms for distributing constraints.

The results of the research objectives will feed into the work carried out to pursue our education objectives: **EO1: to enhance the problem-solving skills and interest in advanced studies** for middle-school to undergraduate students; and **EO2: to enhance the participation of women and Hispanics in computing.**

• **Intellectual Merit.** The success of the Career project will contribute to the NCS and decision-making body of knowledge. The proposed work will advance knowledge about:

i. NCS algorithms: Along with theoretical results about the quality of the solution, the proposed work will address the noise in solution sets, as well as continuums of solutions;
ii. Soft constraints: By developing decision algorithms for over-constrained problems, it will be possible to (semi-)autonomously determine the most relevant soft constraints scheme to solve the original problem;
iii. Constraints distribution over different social groups' structures: the expected communication algorithms will combine theory from speculative constraints with frameworks of social structures to improve the time to solution of given constraints.

• **Broader Impacts.** The project will result in a new Matlab toolbox, expected to enhance the way users approach problem solving and appeal to more users, in particular the Matlab community. The progress and results of the project will be disseminated through the PIs constraintsolving.com website, widely accessed by various communities. Moreover, the PI holds a CoProD workshop that annually brings together NCS researchers and practitioners. The project will take place at UTEP, an institution with more than 75% Hispanic citizens and a leader in the education and advancement of Hispanics. With UTEPs infrastructure and the proposed educational activities, the project will train a new generation of diverse computer scientists with strong problem-solving skills. It will also result in new constraint-based courses, including material that can contribute to a textbook on NCS that will have impact beyond UTEP. Workshops and constraint-oriented hands-on activities will be offered at one middle-school and three high-schools, one of which is a women-only high-school, with potential to inspire students to engage in computing. It is expected that over 450 students will be involved in outreach activities over the course of the grant.

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CAREER: Symbolic-Numeric Constraint-Based Solutions for Real-World Scientific Problems, proposed by PI: Martine Ceberio, University of Texas at El Paso (UTEP)

PROJECT DESCRIPTION

1 Introduction

Numerical constraint solving (NCS) techniques have proven to be efficient to solve real problems ranging from electronic circuit [39] to aircraft design [30]. Yet they are under-utilized. The lack of support of the solvers can partly explain this but most importantly NCS techniques have isolated themselves into a jargon (s.a., constraints, consistencies) and an algorithmic schema (propagation/reduction) that disconnect them, respectively, from real scientific problems and from real needs. Indeed, the field has nurtured many very "constraint-specific" algorithms and needs now to open itself to a more general problem-solving approach and to address real-world scientific needs, such as solving computational intensive problem, over-constrained problems, or handling distributed structures in an efficient and user-friendly manner.

The **goal of this career plan** is to **make NCS better adapted to real-world scientific needs** while **making it more accessible**. The project will broaden the scope and body of knowledge on NCS and the results of the proposed work will be used to introduce students to constraints as a general means for solving problems. The work carried out will use constraints as a basis because they constitute a sound and unambiguous way to state problems and can therefore be used to enhance the problem-solving analytical and critical-thinking skills of students who will be exposed to the project.

In order to achieve its goal, this project will concentrate on theory and algorithms at the crux of the efficiency, adaptability, and distributivity aspects of problem-solving techniques. The following research objectives (RO) will be pursued:

- **RO1 To open NCS to novel techniques, improving scalability.** Our research will be oriented towards designing new algorithms at the junction of local consistency techniques and global constraints: symbolic-numeric approaches will be studied from the theoretical point of view and algorithms will be developed and embedded in a new solving toolbox. New techniques are expected to improve scalability, as well as to open the field to other communities, such as the non-linear programming community;
- **RO2 To assist users facing over-constrained problems**. New methods based on the topology of the search space, in terms of satisfaction of the constraints, will be designed to provide users with alternative solutions; and
- **RO3 To extend the distributed use of NCS** to reflect our emerging networked social structure. The structures of social groups and their potential to support distributed systems will be studied, resulting in new and more complete algorithms for distributing constraints.

The results of the research objectives will feed into the work carried out to pursue our education objectives (EO):

- **EO1 To enhance the problem-solving skills and interest in advanced studies** for middle-school to undergraduate students: the PI will design modules for two undergraduate courses at UTEP as well as a graduate course on problem-solving through constraints; she will also develop hands-on activities for the participating Middle- and High-School of El Paso;

EO2 To increase the participation of women and Hispanics in computing disciplines. Efforts will be spent to expose middle- and high-school students to College activities, research. Workshops will be organized for all middle-school to undergraduate students to present their work together. Mentoring will also take place at all levels of the students involved in this project.

Intellectual Merit. The success of this project will contribute to the NCS body of knowledge and more broadly to the Decision Making community. The proposed work will result in a Matlab toolbox for solving numerical problems and will advance the knowledge about:

• NCS algorithms: Along with theoretical results about the quality of the solution, the proposed work will address the noise in solution sets, as well as continuums of solutions;

• Soft constraints: By developing decision algorithms for over-constrained problems, it will be possible to (semi-)autonomously determine the most relevant soft constraints scheme to solve the original problem;

• Constraints distribution over different social groups' structures: the expected communication algorithms will combine theory from speculative constraints with frameworks of social structures to improve the time to solution of given constraints.

Broader Impacts The project will result in a new Matlab toolbox, expected to enhance the way users approach problem solving and appeal to more users, in particular the Matlab community. The progress and results of the project will be disseminated through the PI's constraintsolving.com website, widely accessed by various communities. Moreover, the PI holds a CoProD workshop that annually brings together NCS researchers and practitioners. The project will take place at UTEP, an institution with more than 75% Hispanic citizens and a leader in the education and advancement of Hispanics. With UTEP's infrastructure and the proposed educational activities, the project will train a new generation of diverse computer scientists with strong problem-solving skills. It will also result in new constraint-based courses, including material that can contribute to a textbook on NCS that will have impact beyond UTEP. Workshops and constraint-oriented hands-on activities will be offered at one middle-school and three high-schools, one of which is a women-only high-school, with potential to inspire students to engage in computing disciplines. It is expected that over 450 students will be involved in outreach activities over the course of the grant.

2 RATIONALE AND BACKGROUND

2.1 Context

Numerical constraint solving (NCS) techniques are important components of intelligent systems. They can efficiently tackle hard continuous, possibly nonlinear problems, such as financial analysis, simulation and synthesis of electronic circuits, failure diagnosis, decision support systems [27], molecular biology [17], geometrical problems [43]. NCS (propagation) techniques made formerly intractable problems very practical: e.g., one famous circuit design problem from Ebers and Moll, formerly solved with a precision of four significant digits in fourteen months using a network of 31 workstations, and now solved in a couple of milliseconds on one workstation thanks to advances in constraint solving algorithms [39].

NCS is different from (constraint) logic programming. Constraint programming (CP) is a declarative way of programming. It can only be related to Prolog in the sense that it is an offspring of it. Constraint Logic Programming (CLP) first extended logic programming by including new kinds of predicates called constraints, which are not necessarily clauses, and are therefore handled differently by the constraint solver. Constraint solvers also extend logic programming solving processes by including solving procedures other than backwards reasoning, in particular constraint propagation. CLP is dedicated to combinatorial, discrete problems. It was extended to Constraint Programming (CP), handling more solving procedures and continuous problems. It is important to note that the field is divided into two main research streams: constraints over discrete domains (leading to work on combinatorial problems: constraint satisfaction), and constraints over continuous domains (leading to mostly work on non-linear problems: constraint solving). NCS is dedicated to the latter.

The power of constraint programming, and **NCS** in particular, has attracted the attention of major companies, s.a. manufacturers Michelin and Dassault, the French railway authority SNCF, airlines Swissair, SAS, and Cathay Pacific, and Hong Kong International Terminals [9]. Nevertheless, NCS has yet to address a lot of challenges, and research in this field is very active. One of them is the lack of knowledge of researchers and domain scientists about constraints in general and their use in decision making processes, making NCS techniques under-utilized in real-world scientific projects.

2.2 Open problems in NCS

Here are presented the major challenges faced in NCS that will be addressed in the proposed project.

Locality of reasonings (LR). Commonly used solving algorithms are based on local consistency algorithms (considering one constraint at a time), and propagation algorithms (propagating information from one constraint to the others) are weak: information is lost (e.g., dependencies between values of variables are lost). Global constraints address this problem for combinatorial problems: there is a need for global continuous constraints as well as for novel more efficient symbolic-numeric algorithms.

Dependency problem of interval computations (DI) (see, e.g., [52] for more information about interval computations). Solvers for NCS problems use intervals to simulate the real line of numbers: all computations are computations over intervals as opposed to computations over real numbers. However, interval computations only provide with an outer approximation of the expected quantities; e.g., over real numbers $x - x = 0$, while for instance $[0,1] - [0,1] = [-1,1] \neq 0$[1]. Therefore, incomplete decisions can only be made and NCS solvers' efficiency is jeopardized.

Handling over-constrained problems (OCP). Although soft constraints are well known, the main challenge when facing over-constrained problems is to know how to relax them / make them soft. There are several (not to say many) frameworks available, e.g., [15, 45]; even when semiring-based constraints are chosen (they encompass the Weighted CSP and other frameworks), many options remain for defining the semiring, distance/quality functions[2], and operators.

Distribution of NCS problems (DNCS). Although distributed constraint programming is a significant and active sub-community of the CP community, there is no technology to either make the best use of, for instance, a multi-core machine, or to adjust to an existing solving architecture of a network.

[1] The set of real intervals is not a group.

[2] The authors mention "values" instead of functions since they consider discrete constraints and users have to assign a value to possible instances (or in some cases, the values are obtained from a priori information, e.g., [58]).

3 RESEARCH PLAN

3.1 RO1: To open NCS to novel techniques, improving scalability

RO1 addresses open problems **LR** and **DI**. Symbolic-numeric approaches are considered: symbolic because the solving techniques are format or syntax specific (e.g., interval evaluations, choice of linear/non-linear techniques); numeric because the constraints are defined on continuous domains and evaluations have to take place at some point in the solving process (even if only little).

3.1.1 Background

Traditionally, **LR** is addressed by either symbolically transforming the set of constraints (triangular form [8, 26], redundant constraints [12, 47, 51]) and then solving them with local consistency techniques, hoping that the locality will be minimized, or by enhancing the local consistency algorithms with heuristics that control the order in which constraints/variables [53] should be considered or the way the search space is explored [29].

Combinatorial constraints that together correspond to a certain schema (e.g., order constraints, timetabling constraints, graph constraints), they can be solved as a global constraint using a dedicated specialized filtering algorithm (informally: an algorithm that will get rid of inconsistent values). For instance, the all-diff constraint is much better solved as a global constraint than as a traditional CSP: solving it with Arc Consistency requires several iterations to reach a solution while the approach using a maximum matching algorithm is much more efficient. Extensive knowledge about global constraints is compiled in the Catalog of Global Constraints [10]. For continuous constraints, the multi-dimensional Newton operator acts as a simple version of a global constraints filtering algorithm and recently [6], a specialized filtering algorithm was proposed for square dense constraint sub-systems. But efforts to extend global constraints to continuous domains need to be done.

Little work has been done to address **DI**; e.g., [41, 25], yet the potential of this research direction was made clear. Indeed, given the branching nature of all solving algorithms [32, 65, 37, 48], better evaluations can increase the rate of box (sub-domain) rejections early in the exploration process and therefore limit the number of tasks to carry out (namely the number of sub-domains to be explored). Research has focused on symbolic transformations of the expressions to be evaluated (evaluations are syntax dependent); e.g., evaluating $(x-1)^2-1$ over intervals is more efficient than evaluating x^2-2x. Unfortunately, although promising, a significant effort has not been put on studying the time complexity of the transformations. Moreover, to date, symbolic transformations do not guarantee consistent evaluation improvement (they depend on the region of evaluation), and the transformations are mostly defined for polynomial expressions only.

3.1.2 PI's Initial Work and Expertise

In the area of NCS algorithms, the PI has extensive experience in developing symbolic-numeric methods. Her work on both LR and DI shows promise and highlights some limitations taken into account for the design of the proposed directions of work of this project.

Symbolic-numeric methods to address LR. The PI addressed LR (and DI as a side effect) by symbolically transforming (non-linear) constraint systems (as if it were linear) using Gaussian elimination [26]. The objectives of her work were:

(1) To decrease the number of non-linear terms in the constraints (the ones being eliminated), because the assumption behind this work was that non-linear terms make the solving process harder and generate multiple occurrences of variables (hence increasing DI);

(2) To transform the original constraint system into a "more triangular" system, hence enhancing the propagation of information: the final system could not be triangular unless originally linear, since non-linear terms were dealt with as new variables, hence making the artificial system rectangular.

The results proved the interest of the method. However, to be fully practical, this work needs to be taken some steps further by considering novel elimination rules: indeed, it often happens that non-linear terms are barely shared between constraints, therefore limiting the power of the proposed algorithm.

One important lesson learned from this work is that, not only symbolic-numeric methods are efficient, but also going farther in the globalization of information (in this case triangularizing) helps reduce the locality of reasoning. This led the PI to orient the research towards the design and study of algorithms at the junction of symbolic-numeric algorithms for local consistency-based algorithms and global constraints.

Symbolic-numeric methods to address DI. New symbolic forms of expressions to be evaluated on intervals were defined and resulted in two main contributions:

(1) Non-linear multivariate expressions of constraints were condensed [24] into a new nested form, in order to limit the number of occurrences of variables (partly responsible for DI). The integration of these new symbolic expressions to NC solvers led to a dramatic decrease of the time to solution. Room for improvement lies in the extension of this work to non-polynomial expressions.

(2) A new symbolic transformation for univariate polynomial expressions resulted into improving the Horner symbolic form. Results showed an improvement of 27% of the width of the interval evaluations when compared against Horner. Multivariate expressions present many more challenges but should be addressed.

This work showed that improving interval evaluations can actually free constraint solvers from the explosion of sub-problems to consider.

Finally, work has been initiated in the area of circular interval arithmetic: the potential for its integration in NC solvers is being studied.

3.1.3 Proposed Work

In order to decrease NC solvers' **<u>time to solution</u>**, new algorithms at the junction of symbolic-numeric techniques for LR and global constraints will be intensively pursued as well as the study and design of ways to integrate different interval arithmetics into NC solvers.

• <u>Global constraints to address LR:</u> **(i)** New **symbolic-numeric algorithms** will be sought that combine/share (at the symbolic level) the information of each (numeric) constraint. Shared sub-expressions will be considered, as well as linear relaxations (in particular, non-square linear system will be studied). In the case of relaxation, the properties of the resulting problem will be studied in order not to loose solutions. Indeed, while discovering new NCS algorithms, it is essential to keep the original properties of completeness of existing NC solvers.

(ii) The use of **non-linear programming and numerical analysis methods** as potential dedicated filtering algorithms for continuous global constraints will be studied and are expected to lead to new global solving techniques. Either classes of continuous global constraints will result or the novel techniques will be used as local method in speed-up pre-processing components of NC solvers. *Moreover, using such methods has the potential to bridge the gap between the NCS and non-linear programming and other communities.*

(iii) Problems at hand might not fit the schema of our global continuous constraints: (1) The possibility of dividing the constraints into sub-problems matching global constraints will be studied. Some variables might need to be left out in order to match a global constraint scheme: consequences on the solution set and the use this approach as an approximation will be studied. (2) Using the results of (i) might help force original problems not matching any global schema to fit into one.

• Interval arithmetics to address DI: New symbolic forms of expressions need to be designed. Additionally, building on the success of circular interval arithmetic [57], new kinds of arithmetic need to be studied as well as their potential integration into NC solvers. Such an endeavor will require the design of new algorithmic schemas in order to continue to guarantee completeness of the results. For instance, covering the search space with disks is not a trivial tasks and either results incomplete or redundant. New algorithmic approach to exploring search spaces need to be discovered. This research direction will be pursued as it can potentially transform search space exploration.

• Besides, other components of the traditional NCS algorithms will be studied for improvement, such as triggers invoking constraints or variables to be checked, in an effort to provide latest technology in the intended released Matlab toolbox. However, these will not constitute the main contribution.

Improved **scalability** is expected to result from the above-mentioned research tasks. However, to further enhance it, specific algorithms for high dimensions and/or hard problems will be studied. In particular, the size of a problem is not the only parameter that makes it hard to solve: the project will (1) study what makes a problem hard in order to define criteria to predict the computing time to solution, and (2) design algorithms based on speculations (by bounding some variables to some sub-domains or even values) that are expected to speed-up the process and cut the computational complexity. This principle is inherited from what is done in timetabling [56, 55] where dimension is high and likeliness to obtain solutions low.

3.1.4 Evaluation and Assessment

The efficiency of the new algorithms will be compared against that of state-of-the-art solvers, such as BARON [59], ICOS [44], Realpaver [38], and ALIAS [62]. They will be tested on benchmarks that are usually used by the NCS community as being problems that exhibit limitations of solvers.

3.2 RO2: To assist users facing over-constrained problems (OCP)

3.2.1 Background

When problems are over-constrained, the usual approach is to relax them, i.e., to make some or all constraints soft. There has been extensive work done in proposing and describing frameworks for soft constraints, such as hierarchical [18, 21, 20, 19], weighted [45, 61], semiring [15, 14] constraint satisfaction problems (CSP), and many others [34, 54, 33, 60]. Semiring CSP were proven to model the other mentioned ones [16]. Recently, although the community still works on defining new solving algorithms, the work on soft constraints has been mostly oriented towards applications.

To date, to the best of our knowledge, there has been no endeavour towards helping users when facing over-constrained problems or, more likely, a possibly over-constrained problem. The objective is to guide users when making constraints soft, and ideally to do this automatically. *This is known as a hard problem and one where a breakthrough would have deep implications for the decision-making community.*

3.2.2 PI's Initial Work and Expertise

The work of the PI in this area has consisted in designing a unifying framework for soft constraints and in applications of soft constraints.

Framework and algorithms. In [11], a unified framework for expressing and solving soft constraints was proposed, based on optimization and NCS methods. This work shows that there is a common ground underlying the concept of soft constraints.

When the PI later designed an expert system to extract knowledge [23] (in the form of constraints or preferences) from users, both soft constraints and Multi-Criteria Decision Making (MCDM) techniques [27] were used. Both concepts are very similar, except that MCDM does not require boundaries on the variables, which is convenient in situations where uncertainty is not controlled.

Applications. The PI also has extensive experience in applying soft constraints to real-world applications. In particular, in her collaboration with Bistarelli, she has been working on an algorithm to solve the Cascade Vulnerability Problem in computer networks [13]. Her experience in applications will be of high importance when the project team studies the properties of different soft constraint framework, and designs methods to guide the user or automate the choice. In particular, her work on soft constraints for camera positioning [22] showed that not all kinds of softness are relevant depending on the problem at hand.

3.2.3 Proposed Work

In order to assist users facing OCP, the research will be based on a framework at the junction of semiring constraints and the framework proposed by the PI in her initial work. Yet, such a framework still offers so many options that help is needed.

(1) The impact of choices of different value sets / semirings and distance functions on the topology of the search space will be studied and theoretical results about the convergence to a given solution set sought. In particular, distances separating the solutions the most while keeping the solving process fast will be sought.

Based on these expected results, it will be possible to offer users alternative solutions/options in terms of softness. Little interaction with the user might be necessary in order to point out constraints that cannot be violated and therefore make better decision as far as the distance functions or regarding the problem to solve. Indeed, a soft constraint problem does not necessarily involve only soft constraints, as shown in [22].

(2) Since soft constraints often involve the optimization of at least one criterion, improvement will also be sought in the area of (multi-objective) optimization algorithms. Namely, the findings from RO1 along with the work of the PI on MCDM will be combined into a faster optimization algorithm.

3.2.4 Evaluation and Assessment

Very little work has been dedicated to continuous soft constraints. Even for combinatorial constraints, the lack of implementation explains the lack of benchmark repository or solver to be compared against. Therefore, in order for the result of this work to be evaluated, the PI and her team will first compare their results against examples that can be found in the soft constraint literature, and will work on creating a benchmark repository. Since the progress of the project will be posted on constraintsolving.com, a discussion forum will be created aiming at gathering users with their problems and at receiving their feedback on the relevance of the proposed soft solutions. This forum is already being created as an outcome of the CoProD'08 workshop.

3.3 RO3: To extend the distributed use of NCS (DNCS)

3.3.1 Background

Researchers in distributed constraint satisfaction constitutes a significant part of the constraint programming community, and they have addressed many problems related to algorithms, communication, complexity. Recently, research has been oriented towards the need for security in distributed systems [50, 7, 67], and towards handling the likeliness of failure in communications [49, 42].

However, nothing was proposed about solvers self-adapting to given networks or architectures. In particular, while there exist compilers that provide support in creating multi-thread applications, such as the Intel C++ Compiler 10.1, no such help exists for multi-threading constraint solving, and even less for adapting to given distributed architectures.

It is our belief that such a help for constraint solving will further enhance the use of constraint solving and in particular NCS techniques. In particular, communication protocols in distributed systems do not make use of emerging social group features. This project aims at studying novel communication algorithms in order to ever increase the time to solution of distributed numerical constraint problems.

3.3.2 PI's Initial Work and Expertise

The experience of the PI in distributed constraints is in **speculative constraint solving**. In [28], a framework for solving constraints distributed over several agents was designed, which made use of speculations to speed up the solving process. The assumption behind this work was that communication is not guaranteed and may fail or experience unpractical delays, and the objective was to obtain a robust-to-failure operational solving procedure. The experimental results showed that the proposed framework indeed sped up the process without overloading the computer's memory for speculations, due to the proposed framework for knowledge revision. Speculations will be included in the research plan as failure in communication has to be taken into account.

3.3.3 Proposed Work

The research aims to automatically distribute NCS problems on given structures while surpassing the performance of local (one thread) NCS solvers. The work carried out will aim:

(1) To automate the process of distributing constraints (i.e., making it adaptable to the current architecture/network) and enhance their solving with speculations, in case communication fails. Latest advances in speculations will be integrated as they now take into account speculative constraints as opposed to speculative values in the PI's initial work.

(2) To study communication protocols existing in networked social structures: social groups' interaction will be studied and mapped to agent communication algorithms; new operational models for computation under such communications will need to be designed.

3.3.4 Evaluation and Assessment

The results will be partially evaluated by comparing the performance in terms of time to solution of the distributed solving process against non-distributed ones. To this end, the solvers mentioned in Subsection 3.3 will be used. Benchmarks will also be designed in order to compare our results (for NC) against a solver for discrete constraints, Disolver [40]. The results will also be evaluated through a statistical analysis of the actual use of each thread of computation and their mutual comparison. Indeed, the aim of the distribution is to be as even in performance as possible, and idle processors are to be avoided.

3.4 Activities Across Objectives

The following activities are part of the research plan but are not specific to one single objective:

(1) A Matlab toolbox will integrate all algorithmic advances achieved through RO1-3. It is expected that such a toolbox will reach out to many more users, in particular the Matlab community.

(2) The CoProD workshop will annually bring together distinguished NCS researchers with current and potential users. Emphasis will be annually put on the areas topical to the advances of research plan to receive feedback from the community on the progress of the project.

4 EDUCATION PLAN

The research component of this Career plan will be integrated and feed its results into the following education plan so as to train the next generation of **diverse computer scientists with strong problem-solving skills** (EO1a), to enhance **interest in advanced studies** (EO1b), as well as to **enhance the recruiting and retention of women and Hispanics** (EO2) into computing disciplines.

4.1 EO1a: To enhance problem-solving skills / Integration of research into education, curriculum development

4.1.1 Context

One of the main barriers to acquiring problem-solving skills lies in the difficulty encountered when modeling the problem itself. Constraints enforce and make such modeling meaningful. The education plan therefore aims at enhancing problem-solving skills using constraint-based material.

Moreover, although there is an increasing need for computer scientist with strong background in nonlinear constraint solving / optimization (e.g., Exxon Mobil, Raytheon, Union Pacific, Honeywell, and NAG emphasize the need for scientists trained in non-linearly constrained problem solving), and that more and more problems involve either purely continuous variables or are mixed problems, there is no trace of courses taught in the United States about complete NCS.

There exist (very few and not current) courses focusing on Constraint Logic Programming, but little mention is made of NCS. There are courses on nonlinear programming (e.g., [2]) that provide extensive knowledge about the traditional Karush-Kuhn-Tucker (KKT) and related methods. However, little (if any) mention is made of the actual practicality of these methods and how they can be implemented safely on computers. In particular, no mention is made of any way to take rounding errors and uncertainty into account while these concerns are, respectively, very practical and very topical.

4.1.2 PI's prior related activities

The PI has experience in teaching NCS courses and modules: she taught Artificial Intelligence (non-required, project-based) and Data Structures (core) over 10 times in the last 6 years and always integrated hands-on activities related to NCS (such as labs related to constraint solving or projects about enhancing part of constraint solvers). She also taught several courses on NCS for graduate students: "Topics in Intelligent Computing on Constraint Programming and their Applications" (CS5354: Summers 2006 and 2008). She also taught a more generally problem-solving-oriented course for undergraduate students: "Topics in Soft Computing on Problem Solving applied to Game Design" (CS4365: Summer 2008).

4.1.3 Proposed work

The education plan aims at enhancing problem-solving skills as early as in El Paso middle and high schools through constraint-related hands-on activities, and to develop the UTEP CS curriculum through undergraduate modules and a graduate course on NCS.

(1) [Grades 8-12] One middle-school and three high-schools in the El Paso area will participate in this project and integrate constraint-based hands-on activities in at least one of their math/science class (e.g., Geometry for Sophomore at Loretto Academy). The hands-on activities will be small problems related to the topics currently taught in class and will mostly consist in modeling problems, which they see in class, as constraint systems to be solved using a simple solver (such as Realpaver [38] in Years 1 and 2 and the new Matlab toolbox in Years 3-5). An expected outcome is increased self-confidence in problem-solving skills and improved analytical and critical-thinking skills.

(2) [UGrad] In order to ensure that students are exposed to problem-solving techniques, course modules will be developed and integrated into two important undergraduate courses:

⋆ *CS3: Data Structures (CS2402)*: The PI taught the course 5 out of the last 6 years. Modules that emphasize problem-solving, and NCS in particular, will be developed.

⋆ *Artificial Intelligence (CS4314)*: Although not a core course, many of UTEP's CS students take this course as an elective (an average of 30 to 35 students usually attend this class). Constraints are part of the covered topics, and it is important to extend the scope of the coverage from simple CLP to NCS to better prepare students to the demands of the workforce. Modules will consist of projects (programming and theoretical).

(3) [Grad] The project will also contribute to graduate curriculum development through the creation of a new course, entitled *Constraint-Based Problem Solving*, that is focused on problem-solving with an emphasis on NCS and techniques discovered in the research activities of this Career plan. This course will be centered on nonlinear NCS and optimization, and will present real-world applications of these techniques. It will be an enhanced and re-focused version of her previously taught courses on NCS. This course is expected to provide students with a stronger background in problem solving and with a deep knowledge of NCS techniques as well as practical issues such as the limitations of computer arithmetic and uncertainty in modeling, and how to cope with these.

(4) [Book] The material developed during activities 1-3, along with research results of the Career project, will contribute toward a textbook on *Symbolic-Numeric Constraint-Based Problem Solving,* for which preliminary contact has been established. Although the text book will likely not be completed during the course of the grant, the PI will work toward creating a draft in the last year of the grant.

4.1.4 Evaluation and Assessment

Activities 1-3 will be monitored through pre- and post-surveys developed and analyzed by the Statistical Consulting Lab of UTEP's Math Department. To evaluate the effectiveness of these activities, the General Linear Mixed Model Analysis will be employed to test differences in the means of the total scores as repeated measurements over 3 time points (baseline, at the end of the semester and six months after the semester). With significance, the Tukey's post-hoc procedure will be used to see where the differences lie. All tests will be conducted at the 0.05 level of significance.

4.2 EO1b: To enhance interest in advanced studies / Mentoring

4.2.1 Context

The PI values students' exposure to research and hands-on activities as well as mentoring. Feedback from the ABET accreditation of the College of Engineering at UTEP in the Fall 2007 showed that undergraduate students desire better interaction with the faculty. Literature shows that students who have more interaction with the faculty are the most likely to be retained and to succeed (e.g., [66, 31, 46]). In particular, in [31], the authors report that "Faculty-student interaction is an essential component of the collegiate experience. Significant research has demonstrated the importance of interaction between faculty members and students, both in and outside of the classroom". In particular, faculty mentoring of undergraduate students proves to be effective on the mentored students' success. In [5], the authors show that the mentored students have a higher GPA (2.45 vs. 2.29), more units completed per semester (9.33 vs. 8.49), and lower dropout rate (14.5% vs. 26.3%).

4.2.2 PI's prior related activities

The PI's has 6 years of experience mentoring and involving (undergraduate and graduate) students in research through her research group, $\mathsf{CR^2G}$: Constraint Research and Reading Group. She also involved undergraduate students in the organization of CoProD'08, resulting in two of them being now involved in research.

4.2.3 Proposed work

Mentoring will be facilitated within the team of the PI (peer-to-peer and faculty-to-student mentoring) and also provided to grade 8-12 students.

(1) The Affinity Research Group (ARG) model will be used. The ARG model [36, 35, 63], funded through the NSF-CISE MII program, defines guidelines for a research environment built on the cooperative learning paradigm and that develops students' team, communication, technical, and research skills. The model comprises student-faculty mentor relationship but also peer-to-peer mentoring. This model was developed to recruit and retain under-represented groups in the computing areas.

(2) Interactions between the students of the team with the students of participating schools will be facilitated to maximize the mentoring experience, as well as the exposure to research of the grade 8-12 students. These students will be given the opportunity to present the work performed at their school at the occasion of an annual workshop bringing together the students of the participating schools with the UTEP ones. They will also be invited to visit and spend time in the research lab of the PI.

4.2.4 Evaluation and Assessment

The effectiveness of the mentoring and exposure to research will be evaluated by an analysis of the students' pre- and post-current-degree decision. For instance, high-school students going to College, UG students going to Grad school, Master's students pursuing a PhD. This data will be collected through UTEP's Center for Institutional Evaluation, Research, and Planning.

4.3 EO2: To enhance the participation of women and Hispanics in computing

4.3.1 Context

The number of women and minorities choosing IT-related undergraduate majors is extremely low, especially in sub-fields such as computer science (CS). According to the CRA's annual Taulbee Survey, only 15% of the bachelor recipients in computer science at research universities were women in 1997-98 [4].

4.3.2 PI's prior involvement

The PI has been strongly involved in several activities to enhance the participation of women and Hispanics in computing; in particular, she has presented at the 2008 Expand Your Horizon conference at UTEP for middle-school girls and is part of two endeavors for increasing the participation of women in computing (BPC 2009).

4.3.3 Proposed work

(1) The project will take place at UTEP, an institution with more than 75% Hispanic citizens and a leader in the education and advancement of **Hispanics**. Moreover, the participating middle and high schools have a vast majority of Hispanic students. The Career activities (e.g., workshops and constraint-oriented hands-on activities) have the potential to inspire students to engage in computing and help retain those already in CS.

(2) One of the participating high-schools, Loretto Academy, is a **women**-only institution. This is a great opportunity for this project, since recruiting women is one of the main challenges of computing disciplines. The other two high-schools are engineering magnet schools. Similar hands-on activities will be provided to all schools. We expect that the experience being carried out at such institutions with such different students will enable us to improve the knowledge about what triggers women's decision not to choose computing careers. The choice of including a middle-school in this project is motivated by the desire to understand the triggers in these communities and hopefully act upon them to increase the participation of women in computing. Moreover, for the purpose of follow-up in our study and analysis, the choice of Hillcrest middle-school and Parkland high-school was motivated by the fact that most students of Hillcrest move on to Parkland, and we hope to be able to get data for the same students over a longer period of time and compare it with national data.

Solely considering numbers, it is expected that over 450 students will be involved in outreach activities over the course of the grant. Since the students being reached are mostly Hispanic and women, a side effect of the above-mentioned activities is also an expected increase in their participation in computing.

(3) Participation in UTEP's outreach programs to women: The project team will continue to be part of the "Expand Your Horizons" conference for middle-school girls at UTEP. Recruiting and retaining women in Engineering is a tricky problem. The Create Plaza at UTEP (see attached letter of its director) supports the WiSE program (Women in Science and Engineering) to "advance the participation of girls and women in STEM fields at all levels through community outreach and professional development activities" [64]. Each year, the project team will identify a successful female researcher (indifferently from the academe or the industry) to be a guest speaker for the WiSE program[3]. The objective will be for this speaker to present how exciting her research/professional activity is, and tie it to her life (since studies have shown that including sociological facts in presentation about science increases the retention and interest of female students). Students of the participating (middle and high) schools, particularly women, will be encouraged to attend.

4.3.4 Evaluation and Assessment

The assessment of this objective will mostly be paired with that of EO1a, through the surveys to be carried by the Statistical Consulting Lab of UTEP.

5 DISSEMINATION PLAN

The PI and her team will inform students, teachers, scientists, and the general community about the progress of the Career plan. The focus of the dissemination plan is to **(1)** raise awareness of the team's efforts and expertise; **(2)** inform the science community of research results and advances in NCS; and **(3)** ensure that scientists have an easy access and use of the results of this project.

In order to achieve the objectives of the dissemination plan, the following activities (DA) will take place:

(DA1). Publication: The team will aim at publishing twice a year at major AI and NCS conferences such as CP, IJCAI, AAAI, and in journals such as Constraints and Computing, as soon as the results reach enough maturity, as well as once a year starting Y2 in educational venues to disseminate the results of the education plan.

(DA2). Organization of annual CoProD workshops: The first edition of CoProD took place in October 2008. Improved through the feedback received at CoProD'08 and following, a CoProD workshop will be organized once a year. By bringing together distinguished researchers and domain scientists, CoProD aims to build a community around a common interest in NCS and more broadly in decision making / problem solving, with an emphasis on transformative algorithms (numeric-symbolic approaches in particular).

(DA3). Integration into Matlab: The new NCS algorithms will be integrated into a new Matlab toolbox, aiming at reaching out to the large Matlab community, comprising many domain scientists, which are a major target for the visibility of NCS and of this project in particular.

(DA4). Online material: The progress of the Career plan will be posted on a dedicated subdomain of the PI's constraintsolving.com: its success (an average of 18 hits a day in 2008) will enhance the visibility of the PI's work. Developed teaching material will be shared through this website as well.

(DA5). Extension of the model to the CAHSI institutions: Leveraging on the fact that UTEP is part of the Computing Alliance for Hispanic Serving Institutions (CAHSI [1]), it is expected that the results of the educational plan can lead to propagating the activities to other institutions

[3]This speaker will also be a speaker at the annual CoProD workshop, held at the same time.

so as to gather more data about keys to enhance the participation of women and Hispanics in computing and share promising practices.

(DA6). Extension of the model to the other Harmony campuses: One of the high schools involved in this project, Harmony Science Academy is part of the Harmony School System comprising over 25 schools in Texas. The PI will then aim at propagating the model of activities of the El Paso campus to others in Texas.

6 MANAGEMENT PLAN

This project will be carried out by the PI and a student team, consisting of 1 PhD, and 2 MS students, as well as students funded through leveraged funding such as REU supplements (undergraduate students) and the Louis Stokes Alliance for Minority Participation (LSAMP [3]) program (PhD students). The activities of each year of the grant are as follows:

Year 1: The PI will build a common knowledge about NCS within the PI's team. In the summer of Y1, the PI will develop her NCS *Constraint-Based Problem-Solving* course. Graduate students will be required to take the course in Fall of Y1, and the undergraduate students will be encouraged to audit the course. Research will focus on RO1 (RO1 will then span on the 5 years), in particular the discovery and development of new symbolic forms for expressions of constraints, as well as the study of global constraints for discrete domains.

In the fall of Y1, the PI will closely work with the pre-college teachers to develop hands-on activities that fit the topics taught in their classes, as well as with the Statistical Consulting Lab at UTEP to design the pre- and post-surveys to be handed out to participating students (data will then be collected and analyzed every semester thereafter). In the spring of Y1, the hands-on activities will be integrated to the targeted pre-college courses and will keep being part of these courses through Y5 and beyond.

Year 2: The definition of several global constraints will be completed and research will begin on RO2. The team will start the integration of the project's advances into a Matlab toolbox at that time. In the summer of Y2, the PI will develop course material for NCS distributed systems to be integrated into her *Constraint-Based Problem Solving* course.

Year 3: The research associated with RO3 will start at Year 3. The PI will offer the course oriented toward distributed systems in Fall of Y3 and will recruit students from those who took this course since special training will be required to embark on RO3. Starting at Y3 and until the end of the project, while continuing their efforts on RO1-2, the PI's team will begin to analyze the first results and impacts of RO1-2, and reconsider the current needs if relevant.

Years 4 and 5: RO3's 2nd activity will be pursued over the last two years of the grant and will lay the foundation about social groups' structures into NCS techniques to be fed into research carried out beyond the 5-year scope of this Career plan.

In parallel to these activities, the team will also **yearly** organize the CoProD workshop at UTEP and a workshop gathering together the pre-college students with her team at UTEP. The constraint-solving.com website will be **monthly** updated with progress and news about the project.

7 SUMMARY: SIGNIFICANCE OF THE PROPOSED WORK

This project aims at making NCS techniques better adapted to real-world needs (mainly: scalable, adaptable, and distributable) and, as a result, change the way problem solving is viewed. The PI is motivated to make NCS techniques more appealing to a broad range of users, through the development of a Matlab toolbox and other approaches and techniques that remove the restrictions of current consistency techniques / propagation algorithms. The NCS community has begun to adopt new techniques, a trend initiated with global constraints (for combinatorial constraints), and this trend needs to be extended to techniques that do not necessarily confine the user to constraint-related jargon. This project is expected to make a difference in the NCS community and to contribute greatly to both the decision-making community, through significant enrichment of the body of knowledge of NCS. In addition, the project will impact the local community through the involvement of the PI's team in strong outreach activities.
The proposed research and education goals align with those of the university in reaching Tier 1 status while providing students from underserved groups access to higher education. The proposed research efforts meet the university's and college's research goals of promoting multi-disciplinary research, and the department's goal of developing a research emphasis in High Assurance Systems. In addition, the course development aligns well with the curriculum goals of the Computer Science Department, and the outreach and student development components meet the department's goal to increase the number of students who enter the undergraduate and graduate programs at UTEP.

8 COMMENTS TO PREVIOUS REVIEWS

My research has matured since the second submission of my Career proposal. As a young investigator, it was important for me to increase my visibility and establish a network of researchers that could provide mentoring. To achieve this, I organized an NCS workshop following SCAN'08 that brought together distinguished NCS researchers and current as well as potential users. In addition, reflecting on the reviewers' comments, I have refined my research by articulating more clearly the global constraints of interests, moving from developing an open source solver to creating a Matlab toolbox, and redefining the scope of my objective on distributed constraints.

9 PI's PRIOR NSF RESULTS

• **NSF BES 0506429:** *SCI Collaborative Research: DAPLDS – a Dynamically Adaptive Protein-Ligand Docking System based on Multi-Scale Modeling.* (Co-PI) PI M. Taufer. $680,152 , 9/2006–1/2008. The objective was to explore the multi-scale nature of algorithmic adaptations in protein-ligand docking and develop computational methods and models that efficiently accommodate these adaptations by means of the immense computing power that can be harnessed through the Internet using public-resource computing. New scheduling algorithms based on constraints were designed.
• **NSF CCF 0839052:** *Constraint Programming and Decision Making Workshop, CoProD'08.* (PI) $5,941, 8/2008–7/2010. The goal of CoProD'08 was to bring together NCS researchers and to address the gap between the great capacity of these techniques and their limited use. CoProD'08 brought together about 30 researchers, from the areas of decision making as well as practitioners. It generated fruitful discussions and enabled some new contacts/collaborations. CoProD's on-going activities consist in developing a website that will engage discussions between decision-making researchers and practitioners and organizing CoProD'09, to take place in 11/09 at UTEP.

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