Repository of Activities (for the instructor's use):

RIDDLES (18)

Objective of the riddles: *keeping the students on their toes with simple and quick problems. Each problem will exhibit a specific aspect of problem solving.*
Such problems as opening and possibly closing (if time allows) will contribute to make the course more lively and students more relaxed and participating.
We should not spend more than 10-15 minutes a day on these.
Most problems are taken from [2].

1/ Six Logicians have finished their dinner together. The waitress comes and asks: "Do you all want coffee?"
The first logician answers: "I don't know."
The second logician answers: "I don't know."
The third logician answers: "I don't know."
The fourth logician answers: "I don't know."
The fifth logician answers: "I don't know."
The last logician answers: "No."
To whom should the waitress serve coffee and why?

2/ Please calculate the expansion of this 26-term expression:
E = (x-a)*(x-b)*(x-c)*…*(x-y)*(x-z).

3/ Mary's mother has exactly four children. The first, a girl, she named Penny; the second she named Nickel and the third she named Dime. Do you know the name of the 4^{th} child?

4/ What is the difference between an old, crumpled and worn ten-dollar bill and a new one?

5/ You are holding a true coin and are told that among five coins on the table, one is counterfeit. The counterfeit is heavier or lighter than a true coin. Find the counterfeit in two weighings using a simple two-pan balance.

6/ You want to cover a chess board with the moves of a knight: starting at the bottom left corner, ending at the top right corner of the board, and never repeating any cell of the board. Is this possible? If so, how? If not, why?

7/ When you add the birth years of a father and his son with each of their ages, what do you obtain?

8/ You have to prepare a cocktail for your guests. The recipe requires 4dl of rum, but you do not have measuring cups. You only have two containers: one of 5dl and one of 3dl. How to measure 4dl with these two containers?

9/ In a square containing 9 X's equally spaced, add two squares (any size) so that it is possible that each X is isolated from the others.

X X X
X X X
X X X

10/ [3] There are 20 gloves in a drawer: 5 pairs of black gloves, 3 pairs of brown, and 2 pairs of gray. You select the gloves in the dark and can check them only after a selection has been made. What is the smallest amount of gloves you need to select to guarantee getting the following?

(a) at least one matching pair
(b) at least one matching pair of each color

11/ [3] A little girl counts from 1 to 1000 using the fingers of her left hand as follows. She starts by calling her thumb 1, the first finger 2, middle finger 3, ring finger 4, and little finger 5. Then she reverses direction, calling the ring finger 6, middle finger 7, the first finger 8, and her thumb 9, after which she calls her first finger 10, and so on. If she continues to count in this manner, on which finger will she stop?

12/ While walking to the library, you cross path with six men, each accompanied by six women. Each woman holds two children. Each child holds three kittens. How many people and pets are going to the library?

13/ A, B, C, and D are placed as follows:

D→C→B→ || ← A

A and B do not see anyone. D sees C and B. C only sees B.
There are 4 hats: 2 black, 2 white. A hat is placed on each of A, B, C, and D's heads without them seeing which color is put on their head. Whoever figures out the color of his or her hat wins.
After a while, one of them says that s/he figured it out? Who is this and what is the color of the hat?

14/ You want to send a package with confidential documents to a friend. In order to protect your package, each of you and your friend has a lock that can be put on the package. The problem is that none of you can have access to the key of the other one's lock. How do you manage so that your package is properly locked and you do not need to exchange keys?
You want to send a package containing confidential documents to a friend. In order to protect your shipment, your friend and yourself each have a lock and the respective key, which can be installed on the package. The problem is that none of you has the key to the other one's lock. How can you lock your shipment and send it without exchanging keys?

15/ You meet three people: Elsa, Julia, and Tony.
Elsa says: “Stephanie has more than 100 spiders at her place!”
Julia answers: “No way! I am sure she has fewer spiders than that!”
Tony adds: “I am sure that she has at least one spider.”
If only one of them says the truth, who is this and how many spiders does Stephanie have at her place?

16/ Two fathers, each accompanied with their own son, go fishing. Each person catches a fish. However, together they only caught 3 fish. Why?

17/ If you had to paint all numbers from 0 to 100 on the doors of the houses of a small village, how many times would you paint number 9?

18/ Train crossing problem. A TGV leaves Paris at 230km/h, an hour later a train leaves Lyon at 150 km/h. When they cross, which train is closer to Paris?

For each of these riddles, the students will be asked to rephrase the problem, to explain how they solved it, and to reflect on what was the tricky aspect to it.
Follow-up activity: for each of the above riddles / small problems, identify a real-life problem that requires a similar approach / reasoning.

REVIEWING SOME SKILLS (6)

LOGIC:
Objective: making them comfortable with manipulating logic statements (necessary to understand problems) and fluent enough that they can play with them while knowing the risks of fallacies.
1/ [1] Suppose that the following statements are both true statements:
p: Chico makes $4.50 an hour.
q: Chico makes more money per week than Terry does.
Find the truth value of each of the following statements:

(a) If Chico earns $3.00 an hour, then he does not make more money per week than Terry does.
(b) If Chico earns $4.50 an hour, then Terry makes more money per week than Chico does.
(c) If Chico makes more money per week than Terry does, then Chico earns $4.50 an hour.
(d) If Chico makes more money per week than Terry does, then Chico earns $5.00 an hour.
(e) If Terry makes more money per week than Chico does, then Chico earns $5.00 an hour.

2/ [1] Suppose that that the following statements are both true statements:
p: Jarvis received a B grade on the final examination.
q: Jarvis passed the course.

Find the truth value of each of the following statements:

(a) If Jarvis received a B on the final, then he passed the course.
(b) If Jarvis received an F on the final, then he failed the course.
(c) If Jarvis received an F on the final, then he passed the course.
(d) If Jarvis passed the course, then he received an A on the final.
(e) If Jarvis failed the course, then he received an A on the final.

3/ Negate these statements and then re-express the results as positive equivalent statements.

(a) Everyone who is majoring in math has a friend who needs help with his homework.
(b) Everyone has a roommate who dislikes everyone.
(c) There is someone in the freshman class who doesn't have a roommate.
(d) Everyone likes someone, but no one likes everyone.

4/ Your enemy has captured you but will give you a chance on the public place to save yourself. A hat will contain two pieces of paper: one that says "death", one that says "life". You will be able to draw one of these pieces of paper. The night before, a spy informs you that your enemy placed two "death" pieces of paper in the hat. What will you do to make sure you survive? (note: you have to pick a piece of paper, and only one piece of paper).

Make the students reflect on what is tricky with logic.
Follow-up activity: Design your own logic puzzle.

ALGEBRA MODELING:

Objective: building students' sense of competency in translating seemingly tricky wording into simple key.

3/ [1] Farmer Gray owns two rectangular fields of the same area. One if 700 yards longer than it is wide; the second is 450 yards shorter than the first, and 400 yards wide. What are the dimensions of each field?

4/ [1] A local charity had set a goal of $70,000 in its fund drive. This morning, when I was solicited for a donation, I asked how the drive was doing. I was told that one third of the amount that had already been collected is equal to three fifths of the amount still needed. How much money is still needed?

Make the students reflect on what they found hard and why?

MORE STRUCTURED PROBLEMS (12)

Objective: having students work on larger problems, but problems that are still very well defined. This set of problems is there to keep testing their ability to understand, and rephrase problems, but with the added objective to train them to resistance to seemingly more complex settings.

FOLLOWING THE CLUES:

1/ [1] An elimination boxing tournament was organized. There were 114 participants and so there were 57 matches in the first round of the tournament. In the second round, the 57 fighters remaining were paired, resulting in 28 matches; one fighter received a bye (that is, did not have to fight in that round). The 29 fighters remaining were then paired, and so on.

(a) How many matches in all were required to determine a winner of the tournament?

(b) (b) How many matches would be required if n people participated in the tournament (where n represents a fixed but unspecified whole number)?

2/ Pages of a book are numbered sequentially starting with 1. If the total number of decimal digits used is equal to 1578, how many pages are there in the book?

3/ A 8 a.m., a train leaves Topeka for Santa Fe and another train leaves Santa Fe for Topeka. The trains maintain constant speeds with no stops. The first train requires five hours to complete the trip and the second train requires seven hours.
At what time do the trains pass each other?

4/ You have two minute glasses: a larger one that allows you to time 7 minutes, a smaller one that allows you to time 4 minutes. Using only these two minutes glasses, how should you proceed to time 9 minutes?

5/ A 8 a.m., a train leaves DC bound for Miami. At the same moment, another train leaves Miami headed to DC. Each train maintains a uniform speed throughout its trip and makes no stops until it arrives at its destination.
If the trains pass each other at 5 p.m. and the train from Miami arrives in Washington DC at 11 p.m., at what time does the train from DC arrive in Miami?

+ Logic and problem understanding

6/ [1] Ms. X, Ms. Y, and Ms. Z – an American woman, an English woman, and a French woman, but not necessarily in this order, were seated around a circular table, paying a game of Hearts. Each passed three cards to the person on her right. Ms. Y passed three hearts to the American. Ms. X passed the queen of spades and two diamonds to the person who passed her cards to the French woman.
Who was the American woman? The English woman? The French woman?

7/ [1] Messrs. Baker, Dyer, Farmer, Glover, and Hosier are seated around a circular table, playing poker. Each gentleman is the namesake of the profession of one of the others. The dyer is seated two places to the left of Mr. Hosier. The baker sits tow places to Mr. Baker's right. The farmer is seated to the left of Mr. Farmer. Mr. Dyer is on the glover's right. What is the name of the dyer?

8/ [1] Stanley Plumb, Bing Cherry, and Walter Mellin decided to form a recording group – The Three Fruits. Not only do they all have wonderful singing voices, but each also plays either the guitar or the banjo.

If Stanley and Walter can both play the guitar, then so can Bing. If Bing cannot play the guitar, then Walter can; but if Walter plays the banjo, then Stanley does not. Either Stanley or Bing, but not both, can play the guitar.
Only one of the three can play both the banjo and the guitar. Which one?

9/ [1] The Turner triplets have an annoying habit – whenever a question is asked of the three of them, two tell the truth and the third lies. When I asked them which of them was born first, they replied as follows:

- Werner: Virna was born first.
- Virna: I am not the oldest.
- Myrna: Werner is the oldest.

Which of the Turner triplets was born first?

10/[1] Adam, Robert, Clifton, Stephen, and Brent are the five starters on the Doylestown Dribblers basketball team. Two are left-handed and three right-handed. Two are over 6 feet tall and three are under 6 feet. Adam and Clifton are of the same handedness, whereas Stephen and Brent use different hands. Robert and Brent are of the same height range, while Clifton and Stephen are in different height ranges. The man who plays center is over 6 feet and is left handed. Who is he?

11/ [3] Four people need to cross a rickety footbridge; they all begin on the same side. It is dark and they have on flashlight. A maximum of two people can cross the bridge at one time. Any party that crosses, either one or two people, must have the flashlight with them. The flashlight must be walked back and forth; it cannot be thrown, for example. Person 1 takes 1 minute to cross the bridge, Person 2 takes 2 minutes, Person 3 takes 5 minutes, and Person 4 takes 10 minutes. A pair must walk together at the rate of the slower person's pace. For example, if person 1 and person 4 walk together, it will take them 10 minutes to cross the bridge. If person 4 returns the flashlight, a total of 20 minutes will have passed.
What is the minimum amount of time that they need to cross the bridge?

12/ [3] You have two couples composed of two men and two women on one side of the river. They all need to cross the river. They only have a small boat to cross. This boat can only carry a maximum of two people at a time.
Now, the two husbands are jealous. As a result, they will not accept that any wife be in the presence of a husband other than hers without the presence of her own husband as well.
How do you proceed to move all four of them from on side of the river to the other? How many trips across the river does it take?

For each of these riddles, the students will be asked to rephrase the problem, to explain how they solved it, to reflect on their approach (was it a satisfactory way to solve it?), and to discuss whether in such problems, there would have been other solutions? How different should these problems have been to exhibit multiple solutions?

Additional activities to follow up on these:
Can you design such a problem? What do you need to think about?
Subsequent challenges: what is the difference between solving a Sudoku and designing one?

LARGER LESS DEFINED PROBLEMS ()

Objective: exposing students to larger, less clearly defined problems, where several solutions exist. Having them reflect of challenges, consequences of solutions, resources that they would need, etc.

1/ **Crossing lights.** You are set to handle the implementation of pedestrian crossing lights at a given intersection. Given a few rules to be respected (*see handout*), you are to:

- Propose at least 2 strategies;
- Discuss each of their pros and cons;
- Lay out a plan of implementation: what resources would you need? Which challenges do you expect to face? Etc.

2/ [4] **Music Deduplication.** You have a bunch of downloaded music tracks, but you also know you've got a lot of duplicates. How would you get rid of the extra copies to save space?

- What if the duplicates aren't 100% identical - e.g., one is in .ogg format and one in .mp3 format?
- What if the duplicates are in the same format, but one is more compressed than the other (quality vs. size tradeoff). Note that you shouldn't just always keep the smaller item, since it might be a partial download, not containing the whole song.

2 bis/ **Image Sorting.** You have stored a lot of images on your computer. Because many names (but not the images) are duplicated, your computer has renamed the duplicates with the extension "-2". How to you arrange your files so that you rename and resort your pictures?

3/ [4] **Logic Puzzles.** There are fairly easy logic puzzles that just involve N-dimensional spaces and finding intersection. Write a solver. E.g., http://www.logic-puzzles.org/

3 bis/ [4] **Sudoku Solver.** How would you go about writing a Sudoku solver? (Very vague description: what else should be asked? What can be understood? What can be done? How do these impact the way the problem will be solved?)

4/ [4] **Disguised Knapsack.** Disguise the knapsack algorithm problem by re-casting it as doing laundry. Can't mix white clothes with dark clothes, but can mix pastel colors with either. Minimize the number of loads of laundry you have to pay for and the number of white shirts you have to replace because they got stained.

5/ [4] **Data Compression.** Discuss what compression actually means. ("Finding things that are the same, and finding more compact ways to represent data than merely repeating the same content.") This has two sub-problems: How do you identify things that are "the same", and how do you encode data so it can be re-expanded. Items for discussion:

- Do items being considered for compression need to be absolutely identical, or can they have a few differences? If you allow differences, how do you decide what's too many? Enough that the encoded representation is larger than not compressing, certainly, but this can hurt your overall compression ratio…
- How do you represent data?
- What if your input is already compressed -- does your data representation add so much overhead that you're substantially worse than not compressing?

Exercise: A cartoon image consists of pixels in a very limited palette of up to 256 colors. Write a compression algorithm. Consider scanning left-to-right, top-to-bottom and combining pixels with identical values. Or perhaps you'd be better off representing rectangles. Or….

6/ [4] **Interns Assignment.** Google has an internal program where we provisionally hire people who didn't quite pass the regular engineering interviews due to lack of skill, not lack of intelligence. We give them an 8-week crash course, then send them on rotation to real work groups. Interns do interviews with prospective teams. After that, interns give a ranked list of teams they'd like to work with, and teams give a ranked list of interns they'd be willing to take on.

Write a program to find the 'best' matches possible. [not in PS1]

This turns out to be a significant mess; there are heuristic approaches, but a strong math background is required to research an algorithm that provably finds at least one "best" fit. Note that this is not quite the same as the Stable Marriage Problem, which happens to strongly disadvantage one group or the other.

GOOGLE PROBLEM (1)

This problem is the one I envision that Mike or Lorne will propose to our students. Any other problem that exhibits similar features will be fine.

Anticipated dates:

- *Presentation of the problem: Friday September 8, at some time between noon and 1:20pm.*
- *Communication with students to touch base and answer follow up questions about the problem: Wednesday September 20, at some time between 9am and 9:50am.*
- *Students' final presentations: Wednesday October 4, 9am to 9:50am.*

This is a smaller version of the classic traveling salesman problem, with modified rules to allow for creative solutions.

Consider a square area in a city that's ten blocks (about a mile) on a side. Within that area, there are ten arbitrary spots on publicly-accessible sidewalks or similar areas. To satisfy the terms of the challenge, each spot must be wet at the same moment.

First challenge: The rules are very very vague. Are they really as vague as they seem, or is there more depth there? E.g., what does "at the same moment" mean - literally all must be wet at the exact same millisecond? How is this checked? Does the time of wetting have to be on-demand, or can we just wait for it to rain? What does "wet" mean - water only, or something like oil that would evaporate only slowly? Does it have to be a single person doing the wetting? Can we just install posts at those locations and let dogs do the watering for us? Can we use drones? How big are the spots? Does the whole spot have to get wet, or just any portion? Lots of opportunities for out-of-the-box thinking.

More challenges:

- Now all the spots need to be wetted on command, within 1 hour, 10 minutes, 1 minute, 1 second. How do your designs change?
- Now all the spots need to be wetted using only one person and the contents of one backpack.
- Now the locations of the spots is not known until the command is issued.
- Now this specific gallon of water must be used.

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[1] Problem Solving Through Recreational Math
[2] Mental Gymnastics, Recreational Mathematics Puzzles
[3] Algorithmic Puzzles
[4] Problems contributed by M. Gainer, Google