LARGE DYNAMIC SYSTEMS:

Towards Predictions of Behavior, ``On-the-fly" Parameter Identification, and mobile device app.

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WHAT IS THE PROBLEM?

Given the model of a dynamical system, use it to make **decisions**...

- What type of decisions?
- What are the challenges? Why is it hard?

Types of decision of interest:

- **Understanding** how the corresponding dynamic phenomenon unfolds under different input parameters: **simulations** $\longrightarrow$ e.g., design decisions
- Based on some prior knowledge of the phenomenon, **predicting** its behavior $\longrightarrow$ e.g., to allow preventive actions for instance
- **Enforcing some behavior**, when control of input or other parameters is possible, and/or recomputing parameters on the fly $\longrightarrow$ e.g., to address an unexpected event

CHALLENGES

- Solving the given dynamical system potentially leads to a **large system of equations** – often nonlinear
  - We can solve very large problems but it takes time
  - Design cases require realizations to make decisions
  - What can be done?
- Let’s add to that the possibility of **uncertainty** in the model, data, etc.
  - Design of armored vehicles (blast computations)
  - Soldiers location and stance are uncertain
- And some interest in **reliability** / guaranteed results... as can be
  - It is not just about getting data to make decisions
  - We’d like to be able to rely on such data

WHY INTERVALS?

INTERVAL COMPUTATIONS VS STOCHASTIC METHODS

$$y' + \frac{x}{\sigma^2} y = 0 \quad y(0) = \frac{1}{\sigma\sqrt{2\pi}} \quad \sigma = [1, \sqrt{3}]$$

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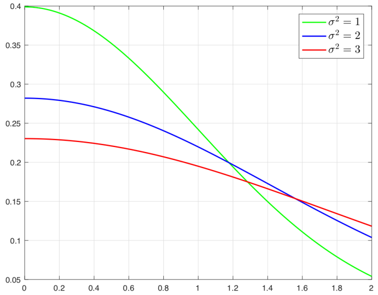

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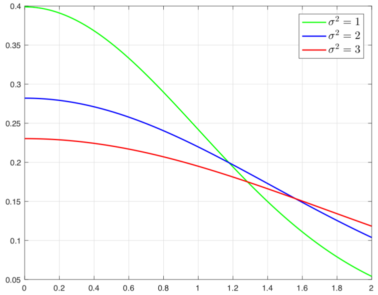

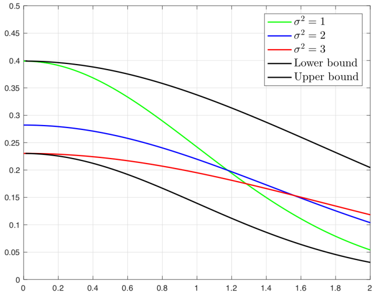

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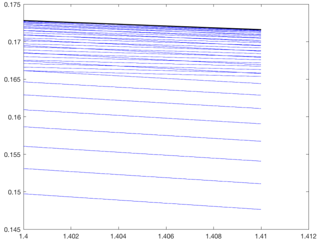

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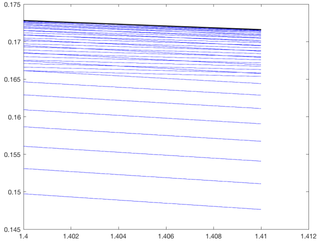
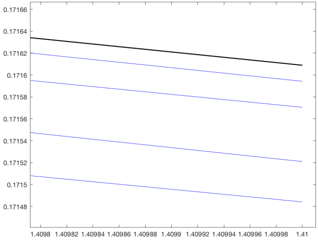




UNCERTAINTY: LOTKA-VOLTERRA

The classic Lotka-Volterra model involves two species: x now stands for the population of the "prey" species, y for the population of the "predator" species.

$$\begin{cases} x' &= \alpha x - \beta xy \\ y' &= -\gamma y + \delta xy \end{cases}$$

α : reproduction rate of prey

β : mortality rate of prey per pred.

γ : mortality rate of predator

δ : reproduction rate of pred. per prey

So where are we in our list of challenges?

- **Large problems**... $\longrightarrow$ are now small
- **Uncertainty**... $\longrightarrow$ can be handled in both FOM and ROM

What are we left with?

- **Predictions:** Given a dynamical system, which is large originally, how can we predict its behavior knowing only a few observations?

PREDICTIONS

So what is the problem?

Given a function:

$$F : \mathbb{R}^p \times \mathbb{R}^n \longrightarrow \mathbb{R}^n; \quad (\lambda, x) \mapsto F(\lambda, x)$$

where, F represents a phenomenon and λ is known. So, $\forall \lambda \in \mathbb{R}$, we solve:

$$F(\lambda, x) = F_\lambda(x) = 0$$

where $x \in \mathbb{R}^n$ is unknown.

Prediction implies that we may not know the parameters λ. However, we have access to some measurements, $Obs = \{x_i, i \in \{1, \dots, n\}\}$ for certain coordinates of x.

$$F(\lambda, x) = 0 \text{ is now: } F_{Obs}(\lambda, x \setminus Obs) = 0$$

Somehow $G(X) = 0$ for some G. However, assume that we have a ROM ϕ for F_λ, we cannot apply it directly to F_{Obs}. So we end up trying to solve:

$$F(\lambda, \Phi x) = 0 \ \wedge \ \forall x_k \in Obs, \ x_k = \sum_i \Phi_{k,i} p_i$$

PREDICTIONS

Here is part of the problem:

$$F(\lambda, \Phi x) = 0 \ \wedge \ \forall x_k \in Obs, \ x_k = \sum_i \Phi_{k,i} p_i$$

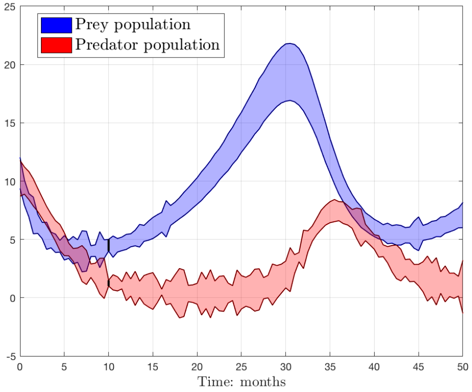
What do we obtain?

- the values of λ that are consistent with the observations on x
- the values of x before and beyond the observations: these allow us to make **predictions**

Let's look at some experimental results

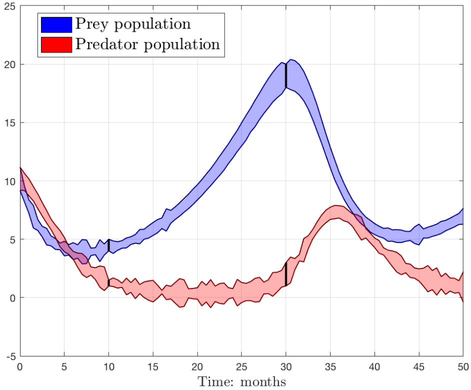
PREYS AND PREDATORS OBSERVED AT $T = 10$

Let's say that we observe: $x(t = 10) = [4, 5]$ and $y(10) = [1, 1.5]$
Let's see what we would obtain with ROM: (21,264 ms)

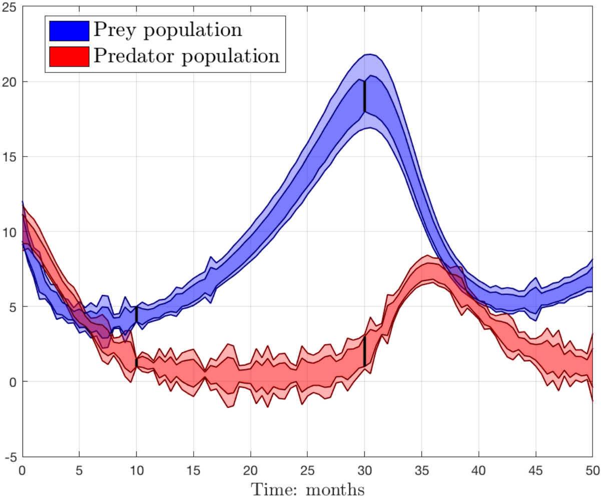


PREYS AND PREDATORS OBSERVED AT $T = 10$ AND 30

Let's say that we observe: $x(10) = [4, 5]$, $y(10) = [1, 1.5]$, $x(30) = [18, 20]$, and and $y(30) = [1, 3]$
Let's see what we would obtain with ROM: (17,453 ms)

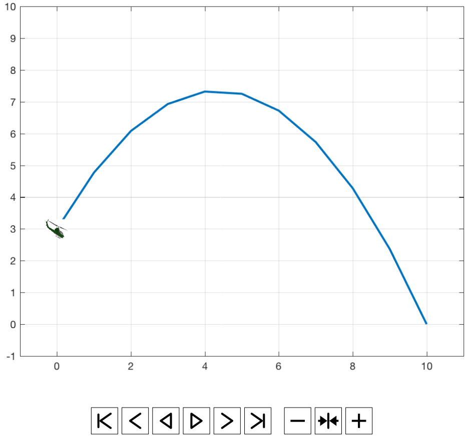


COMPARISON OF PREDICTIONS



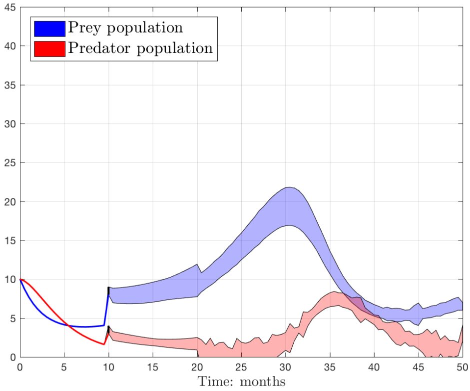
ENFORCING BEHAVIORS: SOME RESULTS

What do we want to do?



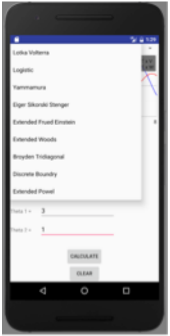
PERTURBATION ON BOTH SPECIES AT $T = 10$

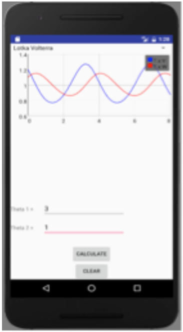
Let's say that a perturbation occurs at: $x(t = 10) = [8.0, 9.0]$ and $y(10) = [3.0, 4.0]$
Let's see what we would obtain with ROM: (21,264 ms)

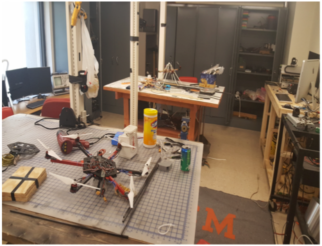


PREDICTIONS: OUR TOOL

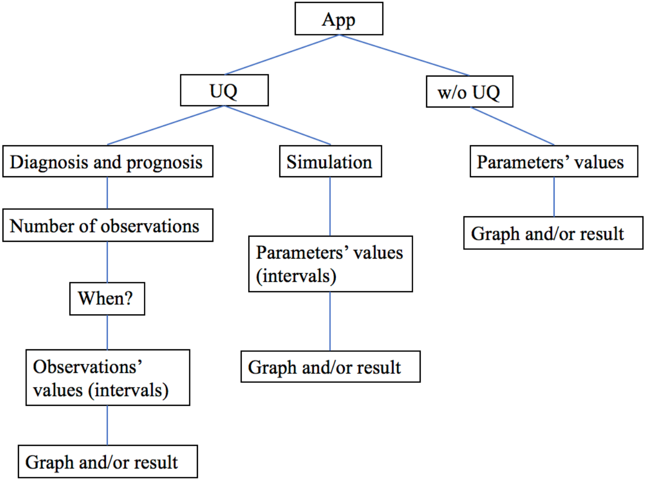
Our goal is to demonstrate the portability of our approach $\longrightarrow$ a tablet ... and a drone?



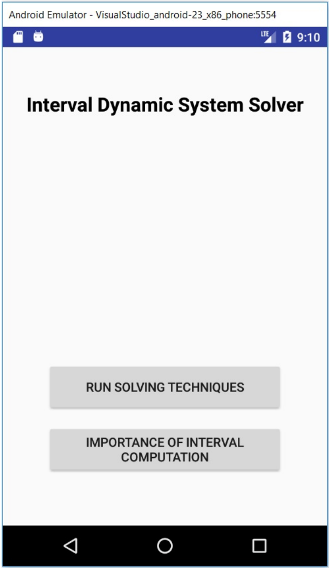




APPLICATION FUNCTIONALITIES



CURRENT WORK



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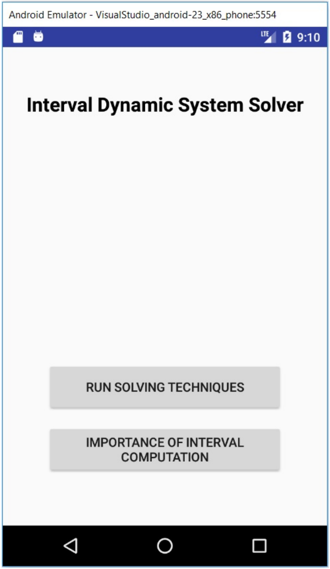

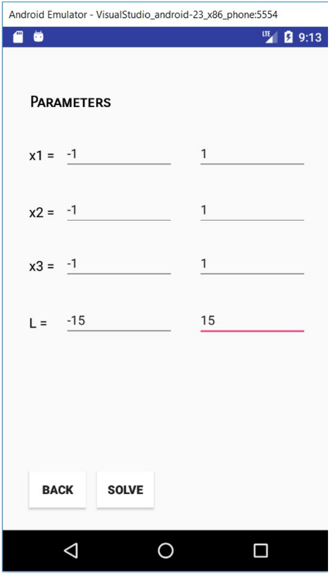

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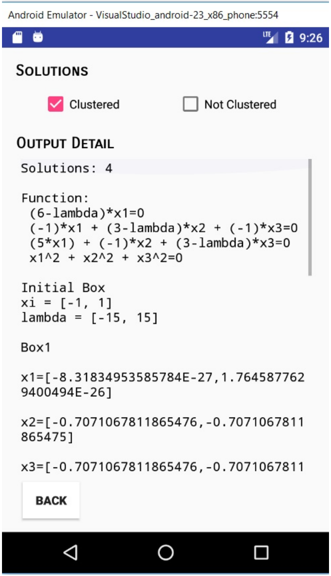

CONCLUSION

- We were able to **enable predictions** even with uncertainty
- We were able to handle uncertainty on a dynamic systems when some unexpected event occurs.
- We were able to do reliable computations on mobile devices

But there is still a lot of work to be done:

- Making this approach practical: outliers, time horizon, computation time, etc.
- Assessing areas in which similar approaches can be taken in truly large systems (that do not need prediction but rather parameter estimation)